A Hybrid Meta-heuristic Method for Optimizing Logistic Networks Subject to Operational Conditions in Practice

Yoshiaki Shimizu¹, Tatsuhiko Sakaguchi², Syota Tsuchiya³
Department of Mechanical Engineering, Toyohashi University of Technology
1-1 Hibarigaoka, Tenpaku-cho, Toyohashi 441-8580, Japan
¹shimizu@me.tut.ac.jp; ²sakaguchi@me.tut.ac.jp

Abstract—Under agile and global manufacturing environment, importance of logistics as a core of supply chain management has been acknowledged increasingly. In this study, we provide a practical hybrid method for a hierarchical logistic network optimization belonging to a tactical level. Since the problem is composed of different kinds of NP-hard combinatorial optimization problems, its rigid solution is almost impossible for real world problems. Hence, to cope with the problem in practice, we have developed a three-level method in terms of the modified insertion method for VRP with time windows. It is deployed based on Ton-Kilo (bilinear of load (ton) and distance (km)) evaluation instead of the conventional Kilo base (distance only). It also includes a new mechanism that enables us to engage in multi-objective analysis between economy and quality of delivery. Validity of the developed method is examined through numerical experiments.

Keywords—Logistics Optimization; Vehicle Routing Problem; Time Windows; Hybrid Method; Insertion Method

I. INTRODUCTION

To improve business efficiency for agile and global manufacturing, importance of logistics as a core of supply chain management has been acknowledged increasingly. Regarding such logistics, let us notice a similarity of hierarchy of decision level, which is popularly classified into long-term, middle-term and short-term levels in production planning. This classification is also expressed by such words as strategic, tactical and operational levels, respectively. Under such understanding, we view the traditional allocation/location problems [1] as the long-term or strategic planning while vehicle routing problems (VRPs) [2] as the short-term or operational planning. Then the middle-term level problem tries to consider a location and vehicle routing at the same time. Traditionally, vast studies have been taken place regarding the strategic level problems (allocation/location). Likewise, we can find a variety of problem formulations at the operational level (VRP) in the last two decades. Among them, in terms of the requirement on certain service, available interval of delivery or time window is popularly discussed. Due to the two-fold aspects like generality and practice required for the strategy and for the operation respectively, however, few studies have been known regarding the middle-term or tactical level concerns. Therefore, this study provides a practical method for the logistic network optimization belonging to the tactical level.

Since the problem at this level is made of different kinds of NP-hard combinatorial optimization problems, its rigid solution is almost impossible for real world problems. Hence, to cope with the problem, we have developed a new hierarchical method so as to solve the problem practically as well as efficiently. Moreover, to commonly evaluate the transportation cost throughout the different levels, we have proposed a modified insertion method that derives a near optimal solution for VRP on a basis of Ton-Kilo (load multiply distances) cost accounting instead of the conventional Kilo basis (only distance). The Ton-Kilo basis is more adequate for cost accounting in practice since we know the transportation cost depend on the load besides the distance. It also provides a new procedure that enables us to engage in multi-objective analysis between economy and quality of delivery represented by the term associated with due dates of delivery. Since thus obtained solution derives only an approximated solution, we further try to improve it by a meta heuristic method such as SA (Simulated Annealing). After all, the developed solution procedure provides a unique integrated method amenable to practical applications at tactical level.

The rest of the paper is organized as follows. In Section II, providing a brief review, we mathematically formulate the problem under consideration. Section III explains the whole scheme of the proposed procedure together with a few key components. We present the results and discussion of numerical experiment in Section IV. Some conclusions are given in Section V.

II. PROBLEM STATEMENTS

A. Brief Review of the Related Studies

Regarding the transportation among the depots and customers, each vehicle must take a circular route from its depot as a starting point and a destination at the same time. This generic problem has been studied popularly as VRP. The VRP is a well-known combinatorial optimization problem, which minimizes the total distance travelled by a fleet of vehicles under various
constraints. Since there are plenty of studies on allocation/location problems, and their formulation is stereotyped in applications, below we reviewed while being interested in those of VRP. On the basis of specific properties of interests, recent studies on VRP aiming at application are able to be classified into the following four kinds.

One of them is an extension from the basic problem that considers only customer demand satisfaction and vehicle payload limit. For example, practical conditions such as customer availability or time window [3, 4], split and mixed deliveries [6] are concerned not only separately but in a combined manner [7]. The second is known as the multi-depot problem that tries to deliver from multiple depots [8]-[10]. The thirds are interested in the multi-objective formulation for the single depot and multi-depot problems [11]-[16]. Though these three classes might belong to the operational level mentioned above, the last one [17]-[20] corresponds to the tactical concern. That is, the decision on the locations of depot is involved besides the VRP.

Due to the difficulty of solution, however, in those studies only small problems with no less than a hundred customers are solved to validate the effectiveness except for the literature [21]. Moreover, they have never noted to account the transportation cost on the Ton-Kilo basis that has been commonly adopted in strategic level. Accordingly, generic and consistent cost accounting has never been given for the tactical problems that apt to concern both with the strategic and operational issues. As sympathized easily, the cost accounting had been better unified by the Ton-Kilo basis due to the reason mentioned already.

**B. Problem Formulation**

Let us consider the logistics network composed of plants, depots and users as depicted in Fig. 1 (a). After the logistic network design has been completed, products are to be delivered from the prescribed plants to every customer via available depots. Transportation between the plants and the opened depots will be done by round trip manner while circular one among depots and customers (See Fig. 1 (b)).

![Fig. 1 Logistics network design problem under consideration](image_url)

Finally, the problem under consideration is formulated as a mixed integer programming problem under two special interests. The first one is a time window or an admissible delivery interval for every customer as a tactical delivery condition that will link to a certain service. Meanwhile, the second one is closely related to the consistency regarding the problem formulation. Conventionally, transportation cost of the allocation problems at the upper level has been accounted on a basis both of distance and load (Ton-Kilo base) while just on distance (Kilo base) in VRP at the lower level. Hence, it becomes necessary to keep consistency of cost accounting when considering the problems over these two levels. Noticing that the Ton-Kilo base is more realistic than the Kilo base since the transportation cost depends not only on distance but also on weight, it makes sense to follow the Ton-Kilo base. Thus formulated problem is not only consistent but also practical in cost accounting. Such idea has never been considered in previous studies.

(p.1) Minimize \( \sum_{i\in I} \sum_{j\in J} c_{k} L_{i,j} F_{i,j} + \sum_{v\in V} \sum_{p\in P} \sum_{p'\in P} c_{p'} d_{p} g_{pp'} z_{pp'} + \sum_{v\in V} F_{v} y_{v} + \sum_{j\in J} F_{p} x_{j} \)

subject to

\[ \sum_{p\in P} z_{kp} \leq 1, \quad \forall k \in K; \forall v \in V \] (1)

\[ \sum_{p\in P} z_{pp'} - \sum_{p'\in P} z_{p'pv} = 0, \quad \forall p \in P; \forall v \in V \] (2)

\[ g_{pp'} \leq W_{v} z_{pp'}, \quad \forall p \in P; \forall p' \in P; \forall v \in V \] (3)

\[ \sum_{j\in J} \sum_{k\in K} z_{jkv} = y_{v}, \quad \forall v \in V \] (4)

\[ \sum_{j\in J} \sum_{k\in K} z_{kvy} = y_{v}, \quad \forall v \in V \] (5)
Here, the objective function is composed of round trip and circular transportation costs and fixed charges of available depots and working vehicles. On the other hand, each constraint means as follows.

Eq. (1): that each vehicle cannot visit every customer twice;
Eq. (2): that coming-in vehicle must leave out;
Eq. (3): upper bound load capacity for each vehicle over every path;
Eqs. (4) and (5): that each vehicle leaves only one depot and return there, respectively;
Eq. (6): that load must be empty for return truck;
Eq. (7): demand satisfaction for every customer;
Eq. (8): mass balance at each depot;
Eq. (9): upper bound capacity at each depot;
Eqs. (10) and (11): upper and lower bounds production at each plant, respectively;
Eq. (12): definition of the arriving time for the next destination;
Eq. (13): that starting time must be within the time window;
Eq. (14): definition of the waiting time;
Eq. (15): specific time limits on each depot;

Integrality conditions and positive conditions are imposed on the respective decision variables.

It is well known that this class of problem becomes NP-hard and extremely difficult to obtain a rigid optimal solution for
real-world size problems. Hence, it is desirable for applications to provide a practical method that can derive a near optimal solution within a reasonable computational effort.

III. A HIERARCHICAL SOLUTION METHOD

A. Basic Procedure

Since it becomes almost impossible to directly solve the resulting problem with real world size, we try to extend our two-level hybrid method for the strategic level problem [22]. The deployed idea is a three-level method as shown in Fig. 2. We employed the modified tabu search for the first level allocation problem. It is a local search controlled by the short memory known as tabu list. Being different from the original method, the modified tabu search also accepts the degraded solution probabilistically in terms of the Maxwell-Boltzmann function like SA [23].

From the above procedure, we can allocate the opening depots among all candidates. Since thus pegged problem refers to a linear program, we transform it into the minimum cost flow problem as shown in Fig. 3 (Transportation cost 1 and cost 2 refer to the \(c_{pd_{ij}}\) and \(c_{id_{ji}}\) in the objective function of (p.1), respectively, and other costs on the edge of MCF graph become all zero.). Then we can apply a certain graph algorithm like RELAX4 [24] to solve the resulting problem extremely fast compared with solving the original LP directly.

The third level VRP problem starts with searching the circular routes based on the result of the second level allocation problem. Letting the customers allocated to each depot be its clients, we can solve the multi-depot VRP problems practically as well as efficiently. The detail of the third level procedure is explained below.

B. Insertion Method for Ton-Kilo Based VRP

Insertion method has been known as an effective approximation method of VRP together with saving method. To cope with the present Ton-Kilo based case, however, we need to extend the original procedure as follows.

Step 1: Select randomly seed customers until the numbers of vehicle.

Step 2: Decide the initial routes by round trip between the depot and each seed customer (Fig. 4(a)).

Step 3: List the remaining customers in the descending order of \(d_{vj}(D_j + q_j)\) for all \(j\) except for the seeds customers, where \(d_{vj}\), \(D_j\) and \(q_j\) denote distance between depot and node \(j\), demand at node \(j\) and own weight of vehicle \(v\), respectively.

Step 4: Select one (say, \(k\)) from the top of the list.

Step 5: Insert customer \(k\) between the node \(i\) and \(j\) for which \(\Delta c_{ij}^k (= \Delta c_{ij})\) becomes minimum for \(\forall i, j\) as long as such insertion will not violate the admissible conditions (capacity or payload of vehicle, for example) (Fig. 4(b)).

\[
\Delta c_{ij}^k = D_1(d_{v1} + d_{v2} + \cdots + d_{v,i-1}) + D_2 + D_3 + q_j(d_{vk} + d_{kj} - d_{v})
\] (16)

Otherwise, go back to Step 4. Then, delete \(k\) from the list.

Step 6: Repeat the above procedures until the list becomes empty (Fig. 4(c)).

Step 7: Letting the above result as an initial guess, improve it based on SA until a certain convergence criterion has been satisfied.
C. Bi-objective VRP Method with Time Window

To deliver products just on time is one of the essential customer service required for the today’s competitive delivery. The idea is deployed as the time window in VRP [25]. As illustrated in Fig. 5, the arrival time of desirable delivery (left) is involved within the time window. In contrast, those of the early and late delivery are outside of the respective time window (middle and right). From the aspect of customer service, the late delivery must not be allowed while the early deliveries may belong to a problem of wasteful time or quality of delivery. Accordingly, besides the payload requirement imposed in Step 5 of the above procedure, we will add another admissible condition (hard condition) whether the arrival time is satisfied or not. On the other hand, regarding the waiting time for the early delivery, we view it as a soft condition that should be achieved as much as possible. Then, the computation in Eq. (16) is modified as Eq. (17).

\[
\Delta_{y}^{k} = \Delta_{c}^{k} + \alpha \cdot \Delta_{w}^{k}
\]

where \(\Delta_{c}^{k}\) denotes the increment of waiting time at node \(j\) when customer \(k\) is inserted between node \(i\) and \(j\). Moreover, \(\alpha\) is a weighting factor between the delivery cost and the waiting time. Generally speaking, since the transport with shorter waiting time needs longer trip distance and accordingly increases the cost, and vice versa, there occurs a trade-off issue between them. By solving the problem with a set of weighting factor \(\alpha\), we can conveniently engage in the trade-off analysis between the economy and the quality of delivery.
IV. NUMERICAL EXPERIMENT

A. General Statement

We carried out a numerical experiment under the computation environment shown in Table I. Table II shows the system parameters employed here. Locations of facilities and system parameters are randomly decided within the prescribed ranges. Regarding the parameters of SA, we set the initial temperature as 100 and annealing factor as 0.9 and repeated the search by 50 times (outer iteration). In each cycle, candidate neighbors are generated through the operations such as insert, swap, cross exchange and 2-opt (See Fig. 6) and this local search is repeated 100 times (inner iteration).

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>COMPUTATION ENVIRONMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS</td>
<td>Debian GNU/Linux 4.0</td>
</tr>
<tr>
<td>Compiler</td>
<td>The GNU C++ compiler 4.1.2 –O3</td>
</tr>
<tr>
<td>CPU</td>
<td>Intel Core2 Duo CPU E6850 3.0(GHz)</td>
</tr>
<tr>
<td>Memory</td>
<td>2(GB)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>EMPLOYED PARAMETER VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant</td>
<td>Max. production [t]</td>
</tr>
<tr>
<td>Depot</td>
<td>Max. capacity [t]</td>
</tr>
<tr>
<td>Customer</td>
<td>Demand [t]</td>
</tr>
<tr>
<td></td>
<td>Time window</td>
</tr>
<tr>
<td></td>
<td>Left end, (e_i)</td>
</tr>
<tr>
<td></td>
<td>Right end, (l_i (l_i\geq(e_i)+60))</td>
</tr>
<tr>
<td></td>
<td>Working hour [h]</td>
</tr>
<tr>
<td>Vehicle</td>
<td>Payload [t]</td>
</tr>
<tr>
<td></td>
<td>Average cruising speed [km/h]</td>
</tr>
<tr>
<td></td>
<td>Transportation unit cost [1/t * km]</td>
</tr>
</tbody>
</table>

To validate the performance, we compared the results among three variants of application as shown in Fig. 7. The first method is the expedient since none of the previous studies consider such a consistency in cost accounting as mentioned in Section II-A. Deriving the circular routes from the conventional or the Kilo-based method of VRP, we re-evaluate its cost in terms of the Ton-Kilo basis and continue the procedure shown thereat. Regarding the variants of the proposed approach, the method 2 skips the additional search at VRP by SA. Hence according to the ascending order of method number, the accuracy will increase in solution.
To evaluate the basic performance of the proposed method, we first solved a small size problem such as $|I|, |J|, |K| = (1, 1, 15)$. This is also solved by the commercial software known as OPL Studio 3.7 (CPLEX9.1) for comparison. In Fig. 8, we compare the results (Pareto front) obtained by solving each problem with various $\alpha$. For example, large $\alpha$ means that short waiting time is more preferable. Hence, each solution for larger $\alpha$ is likely to move in the southeast direction while in the northwest for smaller one. Actually, we can observe the trade-off between the cost and the waiting time. Noticing the profiles of Pareto front of CPLEX and the method 3 are almost identical with each other, we can claim the effectiveness of the proposed method. However, it should be noted that it takes around 6000 s by the CPLEX while only 0.58 s by the proposed method.

Moreover, we examined the results of two cases more in detail. In Tables III and IV, we compare the case when $\alpha=0.1$ and 0.7, respectively. Both cases need 4 vehicles totally, and every one can certainly satisfy the hard constraints such as the payload and the due time. Depending on the weighting factor, the first case ($\alpha=0.1$) requires almost two times of the total waiting time compared with the second one ($\alpha=0.7$). Relying on these results, we claim the proposed method is promising to cope with the real world problems though it is impossible for CPLEX due to the computational load.

<table>
<thead>
<tr>
<th>Root 1</th>
<th>Root 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>cust $k$</td>
<td>demand $</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Total</td>
<td>58</td>
</tr>
</tbody>
</table>
### Table IV Detail of Delivery for Every Root ($\alpha=0.7$)

<table>
<thead>
<tr>
<th>Root 3</th>
<th>Root 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>cust $k$</td>
<td>demand</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>Total</td>
<td>56</td>
</tr>
</tbody>
</table>

### C. Major Result

Here, we prepared the problem whose size is such that $(|J|, |J|, |K|) = (5, 10, 100)$ just like the foregoing case. Then, we repeatedly solved the resulting problem by changing $\alpha$ by 0.01 from 0.1 to 0.7. In Fig. 9, we compare the Pareto fronts only among the three variant methods already mentioned in Fig. 6 since it is impossible to obtain the result by CPLEX. As expected a priori, the method 3 outperforms the others since its Pareto front sticks out most to the direction of the origin of graph. We also notice SA performs poorly as $\alpha$ becomes smaller since the vertical spaces among the plots become narrower. As shown below, this may come from the simple delivery routes for the cost minimization problem in VRP compared with those for the waiting time minimization problem. This simplicity leaves smaller margin when applying SA to update the initial solution. However, computation time of method 3 that involves the additional search by SA (~600 [s]) takes much larger than those of other methods (~50 [s]). Moreover, we can confirm the adequateness of the result if we compare the specified results shown in Fig. 9. The route that weighs the total cost (Fig. 10 (a)) will make the travel distance shorter by serving plain routes. On the other hand, another extreme case (Fig. 10(b)) neglects such effort that attempts to reduce the total waiting time and results in the complicated routes referring to the delivery quality.

![Fig. 9 Comparison among three solution methods](image-url)
To improve business efficiency for agile and global manufacturing, this study has developed a practical method for a logistic network optimization at a tactical level. Thereat, a Ton-Kilo based hybrid method for VRP with time window is developed in terms of the modified insertion method, modified tabu search and SA. It can also concern with multi-objective analysis between economy and quality of delivery. Presenting a general formulation and giving its algorithm, we provided numerical experiments to examine the validity of the proposed approach through comparisons. The first comparison has been done with the commercial software and the second one among the variant methods. Finally, we confirmed the effectiveness of the proposed approach through these experiments. Future studies should be devoted to apply the method to real-world problems concerning with multi-objective optimization and parallel computing [26] to enhance the solution speed for larger problems.

ACKNOWLEDGEMENT

This research was partially supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Exploratory Research, 23656067, 2011-2012, Japan.

NOMENCLATURE

- \( a_p \): arrived time at site \( p \in P \)
- \( c_p \): round trip transportation cost per unit Ton-Kilo value
- \( c_v \): circular transportation cost per Ton-Kilo value of vehicle \( v \in V \)
- \( D_k \): demand of customer \( k \in K \)
- \( d_{pp'} \): path distance between \( p \in P \) and \( p' \in P \)
- \( e_k \): earliest service time or left end of time window for customer \( k \in K \)
- \( F_{pj} \): fixed charge of opening depot \( j \in J \)
- \( F_{Vv} \): fixed charge of vehicle \( v \in V \)
- \( f_{ij} \): shipping amount from plant \( i \in I \) to depot \( j \in J \)
- \( g_{pp'} \): load of vehicle \( v \in V \) on the path from \( p \in P \) to \( p' \in P \)
- \( I \): index set for plant
- \( J \): index set for depot
- \( K \): index set for customer
- \( L_{ij} \): round trip distance between \( i \in I \) and \( j \in J \)
- \( L_{pp'} \): latest service time or right end of time window for customer \( k \in K \)
- \( p, p' \): index set for \( J \cup K \)
- \( q_v \): own weight of vehicle \( v \in V \)
- \( S_{i_{max}} \): maximum production available at plant \( i \in I \)
$S_{min}^*$: minimum production at plant $i \in I$

$s_{pj}$: working time at site $p \in P$

t_{pq}': traveling time between $p \in P$ and $p' \in P$

$U_j$: maximum capacity at depot $j \in J$

$V$: index set for vehicle

$W_v$: maximum capacity of vehicle $v \in V$

$w_k$: waiting time at customer $k \in K$

$s_j=1$ if depot $j \in J$ is available; 0, otherwise

$y_v=1$ if vehicle $v \in V$ is used; 0, otherwise

$z_{pq}':=1$ if vehicle $v \in V$ travels on the path from $p \in P$ to $p' \in P$; 0, otherwise

REFERENCES


Yoshiaki Shimizu is a Professor in Department of Mechanical Engineering, Toyohashi University of Technology, Japan. He received a Doctoral Degree from the Graduate School of Engineering at Kyoto University, Japan in 1982. His teaching and research interests include production and supply chain management, multi-objective optimization and applied operations research.

Tatsuhiko Sakaguchi is an Assistant professor in Department of Mechanical Engineering, Toyohashi University of Technology, Japan. He received a Doctoral Degree from the Graduate School of Engineering at Osaka Prefecture University, Japan in 2004. His teaching and research interests include production and supply chain management, scheduling and machinery processing.

Syota Tsuchiya is a master’s degree student at Toyohashi University of Technology, Japan (2010.4-2011.3). His research interests include production systems engineering and meta-heuristic optimization.