

# The Thermo Economical Cost Minimization of Heat Exchangers

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**Abstract**-A thermo economic optimization analysis is presented yielding simple algebraic formula for estimating the optimum operating temperatures for three different types of heat exchangers which are applied in industrial applications. An economic analysis method is used in the present study, together with the thermodynamical analyses of heat exchangers, for thermo economic optimization of all of the three different types of heat exchanger. The validity of the optimization formulations was checked.

**Keywords**-Thermo economics; Heat exchanger; Cost optimization

## I. INTRODUCTION

Economics of heat exchanger operation is vitally significant. So, optimum operating temperatures for three different types of heat exchanger as shown in Fig. 1 is extremely important in order to have minimum overall life cycle cost for these systems. The optimum values of maximum operating temperatures must be calculated at which minimum cost occurs for four heat exchangers that can be applied for industrial applications for that reason. There exist many parameters for optimizing such heat exchangers in a thermo economical manner. Fixing, and so eliminating all of these thermal and economical parameters depending on the certainty of operating characteristics of applications and the most efficient operating condition of the heat exchangers can determine the optimum operating temperatures for heat exchangers. It is known that the effectiveness of the heat exchanger is directly related to its size together with its initial cost. A thermo economic feasibility study is necessary before installing the heat exchanging systems. The basic topic of the present work depends upon this idea. A new thermo economic optimization technique is realized and presented for this purpose. Original formulae are developed for calculating the optimum operating temperatures at which the minimal total life cycle cost occur. A thorough search of the current literature showed that there was no previous study on minimizing the life cycle cost of a heat exchanger in detail. A well known and practical method,  $P_1$ - $P_2$  method, which is offered by Duffie and Beckman (1980), is used for optimizing the size and operating conditions of heat exchanger, and original interesting results are presented. Variable parameters used in formulating the optimization problem are listed as technical life of the heat exchanger, first cost of the heat exchanger per unit heat transfer area, annual interest rate, present net price of energy, annual energy price escalation rate, annual average operating time, ratio of minimum heat capacity rate into maximum heat capacity rate, design values

of maximum and minimum temperatures of hot and cold fluids for single fluid heat exchanger, overall heat transfer coefficient of the heat exchanger, resale value and the ratio of annual maintenance and operation cost to the first original cost. Optimum steam temperature for single fluid heat exchanger and optimum value of maximum temperature difference for counter current and parallel flow heat exchangers can be calculated easily in a few minutes with the help of practical formulae. A thorough search of the present literature showed that there were several studies about the heat exchangers (Vojtech et. al. 2011, Chung et al. 2002, Grazzini and Rinaldi 2001, Cornelissen and Hirs 1999, Georgiadis 1998, Şahin 1997, Edwards and Matavosian 1982. All of these studies are not directly related to the present work. Original formulae are developed and presented finally.

## II. MATHEMATICAL FORMULATION

### A. Single Fluid ( $C = 0$ ) Heat Exchangers (Steam Heater)

The total cost of single fluid heat exchanger as shown in Fig. 1 can be calculated by using the cost data (Burmeister 1998) as:

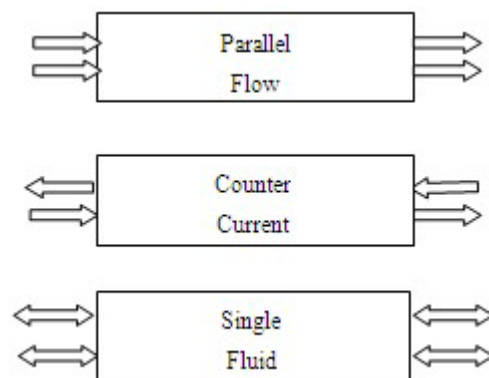


Fig. 1 Schematic Figure of Alternative Heat Exchangers

$$TC = P_2 \cdot C_A \cdot A_{HX} + P_1 \cdot m \cdot C_P \cdot C_{ES} \cdot H \cdot \Delta t \cdot \Delta T \quad (1)$$

The area of heat transfer for single fluid heat exchanger can be calculated by the following.

$$A_{HX} = -\frac{m \cdot C_P}{U} \cdot \ln(1 - \varepsilon) \quad (2)$$

Where the effectiveness of the heat exchanger is defined by:

$$\epsilon = \frac{\Delta T}{T_S - T_{\min}} = \frac{T_{\max} - T_{\min}}{T_S - T_{\min}} \quad (3)$$

The total cost of the single fluid type of heat exchanger per unit heat capacity rate can be estimated by means of the following function.

$$\frac{TC}{m.C_p} = -\frac{P_2.C_A}{U} \cdot \ln\left(1 - \frac{\Delta T}{T_S - T_{\min}}\right) + P_1.C_{ES} \cdot H.\Delta T.\Delta t = -A \cdot \ln\left(\frac{T_S - T_{\max}}{T_S - T_{\min}}\right) + B.\Delta T.T_S \quad (4)$$

where

$$A = \frac{P_2.C_A}{U} \quad (5)$$

and

$$B = \frac{P_1.H.\Delta t}{4} \quad (6)$$

The cost of energy in the form of steam as a function of steam temperature in dollars per joule can be approximated (Stoecker1989) as in the following form.

$$C_{ES} = 10^{-11} \cdot \frac{T_S}{4} \quad (7)$$

And also:

$$1 - \epsilon = 1 - \frac{\Delta T}{T_S - T_{\min}} = \frac{T_S - T_{\max}}{T_S - T_{\min}} \quad (8)$$

The first derivative of the total cost function with respect to steam temperature can be obtained for optimization purpose by the following equality.

$$\frac{\partial[TC/(m.C_p)]}{\partial T_S} = \frac{-A.\Delta T}{(T_S - T_{\min}).(T_S - T_{\max})} + B.\Delta T = 0 \quad (9)$$

Eq. (9) can be simplified to form Eq. (10) as:

$$T_S^2 - (T_{\max} + T_{\min}).T_S + T_{\max}.T_{\min} - \frac{A}{B} = 0 \quad (10)$$

The optimum operating steam temperature can be calculated by the help of Eq. (11).

$$T_{S,opt} = \frac{T_{\min} + T_{\max} \pm \sqrt{(T_S + T_{\max})^2 - 4.(T_{\max}.T_{\min} - A/B)}}{2} \quad (11)$$

The second derivative of the total cost function is always positive which indicates a local minimum certainly.

$$\frac{\partial^2[TC/(m.C_p)]}{\partial T_S^2} = \frac{A.\Delta T.[2.T_{S,opt} - (T_{\max} + T_{\min})]}{(T_{S,opt} - T_{\min})^2.(T_{S,opt} - T_{\max})^2} > 0 \quad (12)$$

Since:

$$T_{S,opt} > (T_{\max} + T_{\min})/2 \quad (13)$$

### B. Counter Current (C = 1) Heat Exchangers

The same procedure can be applied for the counter and parallel flow heat exchangers and the following equations can be obtained. The cost of energy is approximated due to the maximum operating temperature difference, in \$/J (Stoecker1989) also as in Eq. (14).

$$C_{EN} = 10^{-11} \cdot \frac{\Delta T_{\max}}{4} \quad (14)$$

$$\epsilon = \frac{NTU}{NTU + 1} = \frac{U.A_{HX}/(m.C_p)}{1 + [U.A_{HX}/(m.C_p)]} \Rightarrow A_{HX} = \frac{m.C_p \cdot \epsilon}{U.(1 - \epsilon)} \quad (15)$$

$$\frac{TC}{m.C_p} = \frac{P_2.C_A}{U} \cdot \frac{\epsilon}{(1 - \epsilon)} + P_1.C_{EN} \cdot$$

$$H.\Delta T_{\max}.\Delta t = A \cdot \left(\frac{\Delta T / \Delta T_{\max}}{1 - \Delta T / \Delta T_{\max}}\right) + B.\Delta T_{\max} \quad (16)$$

$$A = \frac{P_2.C_A}{U} \quad (17)$$

$$B = \frac{P_1.H.\Delta T.\Delta t}{4} \quad (18)$$

$$\epsilon = \frac{\Delta T}{\Delta T_{\max}} \quad (19)$$

$$\frac{\partial[TC/(m.C_p)]}{\partial(\Delta T_{\max})} = \frac{-A.\Delta T}{(\Delta T_{\max} - \Delta T)^2} + B = 0 \quad (20)$$

$$\Delta T_{\max}^2 - 2.\Delta T.\Delta T_{\max} + \Delta T^2 - \frac{A}{B}.\Delta T = 0 \quad (21)$$

$$\Delta T_{opt} = \frac{2.\Delta T \pm \sqrt{(2.\Delta T)^2 - 4.(\Delta T^2 - (A/B).\Delta T)}}{2} = \Delta T + \sqrt{A.\Delta T / B} \quad (22)$$

$$\frac{\partial^2 [TC / (m.C_p)]}{\partial T_{max}^2} = \frac{2.A.\Delta T}{(\Delta T_{opt} - \Delta T)^3} > 0 \quad (23)$$

$$\Delta T_{opt} > \Delta T \quad (24)$$

C. Parallel Flow Heat Exchangers

$$A_{HX} = -\frac{m.C_p}{U} \cdot \ln[1 - \varepsilon.(1 + C)] \quad (25)$$

$$\frac{TC}{m.C_p} = -\frac{P_2.C_A}{U.(1 + C)} \cdot \ln \left[ 1 - \frac{(1 + C).\Delta T}{\Delta T_{max}} \right] + P_1.C_{EN} \cdot$$

$$H.\Delta T.\Delta t = \frac{-A}{1 + C} \cdot \ln \left( 1 - \frac{(1 + C).\Delta T}{\Delta T_{max}} \right) + B.\Delta T_{max} \quad (26)$$

$$C_{EN} = 10^{-11} \cdot \frac{\Delta T_{max}}{4} \quad (27)$$

$$B = \frac{P_1.H.\Delta T.\Delta t}{4} \quad (28)$$

$$A = \frac{P_2.C_A}{U} \quad (29)$$

$$\varepsilon = \frac{\Delta T}{\Delta T_{max}} \quad (30)$$

$$\frac{\partial [TC / (m.C_p)]}{\partial (\Delta T_{max})} = \frac{-A.(1 + C).\Delta T}{\Delta T_{max}^2 - (1 + C).\Delta T_{max}} + B = 0 \quad (31)$$

$$\Delta T_{max}^2 - (1 + C).\Delta T.\Delta T_{max} - \frac{A}{B}.\Delta T = 0 \quad (32)$$

$$\Delta T_{opt} = \frac{(1 + C).\Delta T \pm \sqrt{(1 + C)^2.\Delta T^2 + 4.(A/B).\Delta T}}{2} \quad (33)$$

$$\frac{\partial^2 [TC / (m.C_p)]}{\partial T_{max}^2} = \frac{A.\Delta T.[2.\Delta T_{opt} - (1 + C).\Delta T]}{(\Delta T_{opt}^2 - (1 + C).\Delta T.\Delta T_{opt})^2} > 0 \quad (34)$$

$$\Delta T_{opt} > (1 + C).\Delta T / 2 \quad (35)$$

D. Counter Current (C # 1) Heat Exchangers

$$\varepsilon = \frac{1 - e^{-NTU(C-1)}}{1 - C.e^{-NTU(C-1)}} \Rightarrow A_{HX} = \frac{-m.C_p}{U.(1 - C)} \cdot \ln \left( \frac{1 - \varepsilon}{1 - C.\varepsilon} \right) \quad (36)$$

$$\frac{TC}{m.C_p} = -\frac{P_2.C_A}{U.(1 - C)} \cdot \ln \left[ \frac{1 - \varepsilon}{1 - C.\varepsilon} \right] + P_1.C_{EN} \cdot$$

$$H.\Delta T.\Delta t = \frac{-A}{1 - C} \cdot \ln \left( 1 - \frac{(1 + C).\Delta T}{\Delta T_{max}} \right) + B.\Delta T_{max} \quad (37)$$

$$C_{EN} = 10^{-11} \cdot \frac{\Delta T_{max}}{4} \quad (38)$$

$$B = \frac{P_1.H.\Delta T.\Delta t}{4} \quad (39)$$

$$A = \frac{P_2.C_A}{U} \quad (40)$$

$$\varepsilon = \frac{\Delta T}{\Delta T_{max}} \quad (41)$$

$$\frac{\partial [TC / (m.C_p)]}{\partial (\Delta T_{max})} = \frac{-A.\Delta T}{(\Delta T_{max} - C.\Delta T).( \Delta T_{max} - \Delta T)} + B = 0 \quad (42)$$

$$\Delta T_{max}^2 - (1 + C).\Delta T.\Delta T_{max} - \frac{A}{B}.\Delta T + C.\Delta T^2 = 0 \quad (43)$$

$$\Delta T_{opt} = \frac{(1 + C).\Delta T \pm \sqrt{(1 + C)^2.\Delta T^2 - 4.[C.\Delta T^2 - (A/B).\Delta T]}}{2} \quad (44)$$

$$\frac{\partial^2 [TC / (m.C_p)]}{\partial T_{max}^2} = \frac{A.\Delta T.[2.\Delta T_{opt} - (1 + C).\Delta T]}{(\Delta T_{opt} - C.\Delta T)^2.( \Delta T_{opt} - \Delta T)^2} > 0 \quad (45)$$

$$\Delta T_{opt} > (1 + C).\Delta T / 2 \quad (46)$$

The economic parameters P<sub>1</sub> and P<sub>2</sub> are defined as in following equations (Duffie and Beckman 1980). If I is equal to d:

$$P_1 = \frac{N}{1 + i} \quad (47)$$

And for i # d:

$$P_1 = \frac{1}{(d - i)} \left\{ 1 - \left[ \frac{1 + i}{1 + d} \right]^N \right\} \quad (48)$$

And

$$P_2 = P_1.M_s - R_v.(1 + d)^{-N} \quad (49)$$

III. RESULTS AND DISCUSSION

For a typical heat exchanger problem, it is assumed that  $i = d = 0.08$ ,  $U = 500 \text{ W}/(\text{m}^2 \cdot \text{K})$ ,  $H = 2000 \text{ hr}/\text{yr}$ ,  $N = 20 \text{ yr}$ ,  $T = 200 \text{ K}$  for counter and parallel flow heat exchangers,  $T_{\text{max}} = 250\text{C}$  for single fluid heat exchanger,  $\Delta T_{\text{min}} = 50\text{C}$ ,  $C_A = 100 \text{ } \$/\text{m}^2$ ,  $M_s = 0$ ,  $R_v = 0$ . The values of total costs for this specific example are depicted in Figs. 2 to 5. There exist specific local minimum cost points in each figure. The best maximum steam temperature is calculated for single fluid heat exchanger as 253 degrees C by using Eqn. (11) whereas optimum maximum temperature difference values are calculated by using Eqns. (22), (33) and (44) as 224, 403 and 206 degrees C respectively for counter flow  $C = 1$ , parallel flow  $C = 1$  and counter flow  $C \neq 1$  heat exchangers. The values of optimum temperatures with corresponding effectiveness and heat transfer area values are presented in Table 1. The same data was used for all of the four alternative heat exchangers.  $C$  is selected as unity for parallel flow heat exchanger and 0.5 as an example for counter flow heat exchanger for  $C \neq 1$  case as illustrated in concerning figures. There exist so many design tools such as computer codes for designing the heat exchangers. These codes are helpful for speedy estimation of heat exchangers and they are used in industry. These computer codes select the standardized heat exchanger due to predesigned operating temperature levels by using thermo-hydraulic data without considering economics.

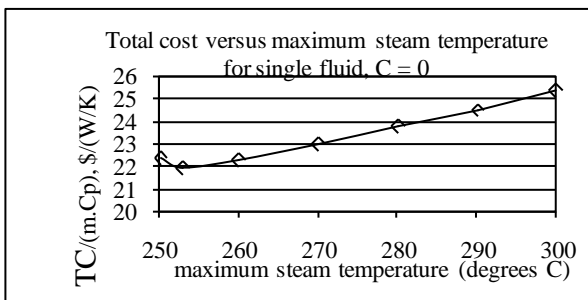


Fig. 2 Total Cost versus Steam Temperature for Single Fluid Heat Exchangers.

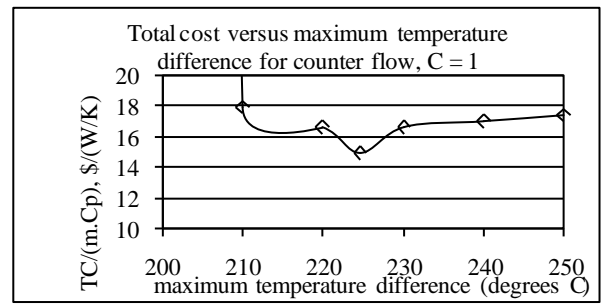


Fig. 3 Total Cost versus Maximum Temperature for Counter Current Heat Exchangers  $C = 1$ .

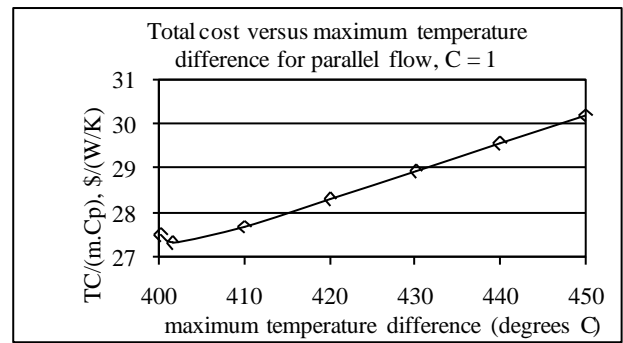


Fig. 4 Total Cost versus Maximum Temperature for Parallel Flow Heat Exchangers  $C = 1$ .

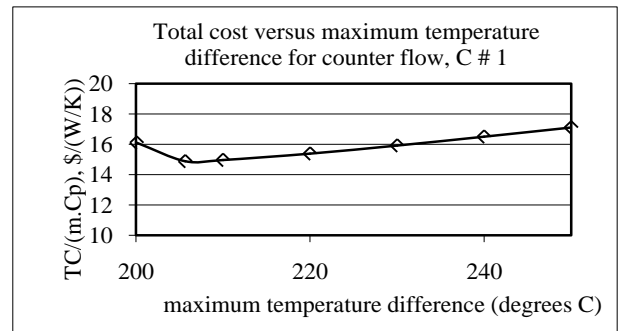


Fig. 5 Total Cost versus Maximum Temperature for Counter Current Heat Exchangers  $C \neq 1$

TABLE 1 THERMO ECONOMIC PERFORMANCE VALUES FOR FOUR HEAT EXCHANGERS.

Type of heat exchanger	Single fluid, $C=0$	Counter, $C=1$	Parallel, $C=1$	Counter, $C \neq 1$
$T_{s,\text{opt}}$ or $\Delta T_{\text{max,opt}}$ (C)	252.96	224.50	401.49	205.67
Optimum effectiveness	0.985	0.891	0.498	0.972
$A_{\text{HX}}/(\text{m.Cp})$ ( $\text{m}^2 \cdot \text{K}/\text{W}$ )	0.0042	0.0082	0.0055	0.0058
$\text{TC}/(\text{m.Cp})$ [ $\$/(\text{W}/\text{K})$ ]	21.925	14.966	27.326	14.882

IV. CONCLUSION

It can be deduced that there exists always a local minimum value in heat exchanger applications for three alternative configurations. Excessive steam temperature or maximum temperature difference will not be cost effective beyond the optimum values in spite of a greater heat transfer

recovery potential. The maximum operating temperature of heat exchanging fluid has dominant effect on the amount of total life cycle cost. It is clear that there exist good thermal performance at the optimum point for each alternative heat exchanger. These types of systems must be designed close to this optimum point. The present formulae may seem to be helpful for heat exchanger designers and manufacturers.

## REFERENCES

- [1] Burmeister LC. 1998. Equipment cost estimation. In Elements of Thermal-Fluid System Design, Prentice Hall: New Jersey; 263-270.
- [2] Chung K, Lee K, Kim W. 2002. Optimization of the design factors for thermal performance of a parallel-flow heat exchanger, International Journal of Heat and Mass Transfer. 45(24), 4773-4780.
- [3] Cornelliessen RL, Hirs GG. 1999. Thermodynamic optimization of a heat exchanger, International Journal of Heat and Mass Transfer. 42(5), 951-959.
- [4] Duffie JA, Beckman WA. 1980. Solar process economics. In Solar Engineering of Thermal Processes, Wiley: New York; 376-407.
- [5] Edwards DK, Matavosian R. 1982. Thermoeconomically optimum counter flow heat exchanger effectiveness, Journal of Heat Transfer-Transactions of the ASME. 104(1), 191-193.
- [6] Georgiadis MC, Rotstein GE, and Macchietto S. 1998. Optimal design and operation of heat exchangers under milk fouling, AIChE Journal. 44(9), 2099-2111.
- [7] Grazzini G, Rinaldi R. 2001. Thermodynamic optimal design of heat exchangers for an irreversible refrigerator, International Journal of Thermal Sciences. 40(2): 173-180.
- [8] Stoecker WF. 1989. Design of Thermal Systems, 3<sup>rd</sup> Edn, Mc Graw-Hill: New York; 468.
- [9] Şahin AZ. 1997. Thermodynamic design optimization of a heat recuperator, International Communications in Heat and Mass Transfer. 24(7), 1029-1938.
- [10] Vojtech,T et.al. 2011. Optimum design of fluid distribution systems in heat exchangers, Asia Pasific Journal of Chemical Engineering. 6(5), 750-759.

## APPENDIX

A Constant depending on values of fixed operating parameters as defined in Eqns. (5), (17), (29), (40)

AHX Area of heat exchanger, (m<sup>2</sup>)

B Constant depending on values of fixed operating parameters as defined in Eqns. (6), (18), (28), (39)

C Ratio of minimum to maximum heat capacity rates of two streams in heat exchanger

CA Area dependent first cost of the heat exchanger, (\$/m<sup>2</sup>)

C<sub>EN</sub> Cost of energy depending on the maximum temperature, (\$/J)

C<sub>ES</sub> Cost of energy in the form of steam, (\$/J)

C<sub>max</sub> Higher heat capacity rate in heat exchanger, (= mC<sub>max</sub>), (W/K)

C<sub>min</sub> Lower heat capacity rate in heat exchanger, (= mC<sub>min</sub>), (W/K)

C<sub>p</sub> Specific heat of circulating fluid having minimum heat capacity rate, [ J/ (kg.K)]

d Market discount rate in fraction

H Annual time of operation, (h/yr)

i Energy price escalation rate in fraction

m Mass rate of flow of circulating fluid having minimum heat capacity rate, (kg/s)

M<sub>s</sub> Ratio of annual maintenance and operation cost into first original cost

N Technical life, (yr)

NTU Number of transfer units

P<sub>1</sub> Ratio of the life cycle energy cost savings to the first year energy cost savings, (yr)

P<sub>2</sub> Ratio of the life cycle expenditures incurred because of the additional capital investment to the initial investment

R<sub>v</sub> Ratio of resale value into the first original cost

T<sub>max</sub> Maximum temperature of hot fluid at the inlet side of single fluid heat exchanger, (C)

T<sub>min</sub> Minimum temperature of cold fluid at the inlet side of single fluid heat exchanger, (C)

T<sub>S</sub> Temperature of phase changing steam in single fluid heat exchanger, (C)

T<sub>S,opt</sub> Optimum temperature of steam in single fluid heat exchanger, (C)

TC Total cost of the heat exchanger

U Overall heat transfer coefficient, [W/ (m<sup>2</sup>.K)]

Δt Number of seconds in an hour, (= 3600 )

ΔT Inlet-exit temperature difference for fluid having minimum capacity rate in counter or parallel flow heat exchangers, (C)

ΔT<sub>max</sub> Maximum temperature difference between hot and cold fluid inlet in counter or parallel flow heat exchangers, (C)

ΔT<sub>opt</sub> Optimal maximum temperature difference between hot and cold fluid inlet in counter or parallel flow heat exchangers, (C)

ε Effectiveness of heat exchanger



Dr. M. Sait Söylemez was born in Gaziantep, Turkey, in 1962. He got his BS and MS degrees in Mechanical Engineering in 1985 and in 1988; he obtained his MS degree in Mechanical Engineering Department of the Middle East Technical University, Turkey. Then he continued his studies and got his Ph. D. Degree at the Mechanical Engineering Department of the University of Gaziantep, in Turkey, in 1992. He has published over 50 research papers in the International Journals and conference proceedings. He is currently working as Lecturer in the same university. His major interests are heat transfer and energy economics.