An Algorithm for Optimal Load Dispatch in a Power System Incorporating Transmission Cost

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***Abstract*-The paper describes an algorithm for assigning transmission cost to generator(s) and load(s) based on sensitivity relation between line power flow and power generation from generating stations and loads. It allows formulation of cost function for generating station containing cost co-efficient representing transmission line power flow assigned to generating stations. As a result, solution of co-ordination equation for determining optimum generation scheduling involves transmission cost pertaining to the generating station. This ensures better co-ordination of transmission cost in optimal generation scheduling and reduction in transmission cost. Further, it allows assigning of transmission cost to the loads that are drawing power through the line.**

N = Total number of buses in the system.

NG = Total number of generation buses.

NL = Total number of load buses.

NT = Total number of lines considered with transmission cost.

Pi = Injected active power at ith bus.

Vi = Magnitude of voltage at ith bus.

δi = Angle of the bus voltage at ith bus.

PGi = Active power generated at ith bus.

PDi = Active power demand at ith bus.

PGT = Total generation in MW.

PDT = Total load demand in MW.

PL = Total system loss.

r_{km} + jx_{km}**= The series impedance of the line connected between kth and mth buses.**

y(c)km = Half line charging of line betweenkth and mth buses.

Keywords-Power System Planning, Optimal Load Dispatch, Sensitivity Factor, Transmission Cost

I. INTRODUCTION

The optimal load dispatch is used to describe a wide spectrum of problem in planning and operation of a power system in which mode of planning and operation is decided by the status of generations, demands and system loss. The problem of optimum load dispatch in a power system is solved through an iterative procedure involving the solution of co-ordination equations and followed by a load flow analysis until the limiting criterion of total generation minus loss is equal to total load demand is satisfied. For the solution of co-ordination equation, incremental system loss is required for the generating stations. Kirchmayer [1] represented system loss using loss co-efficient as PL = $[PG]^T$ [B] [PG], where, [PG] = real power generation matrix and [B] = loss coefficient matrix. Carpentier [2] formulated a new technique in which general problem of minimizing the instantaneous operating cost of a power system is subjected to both network constraints and inequality limits. A basic direct-search maximization method was proposed by Smith and Tong to minimize system losses with respect to reactive powers at the buses [3]. Dopazo and et al presented a method for optimization of both real and reactive powers [4]. For determination of reactive power scheduling a gradient method was used with an arbitrary power and it was improved from iteration to iteration using gradient algorithm. El-Abiad and Jaimes [5] suggested a method similar to that of Tinney and Domnel [6]. A distinguishing feature of the El-Abiad and James method is that both Qi and |Vi| are monitored at each iteration. However the influence of δi on reactive power is neglected for the purpose of economic scheduling. Elgerd[7] suggested a method for real power optimal load dispatch in which Incremental Transmission Loss (ITL) for the generator buses are calculated directly using Jacobian matrix of load flow analysis and the ITL for the slack bus is taken as zero. Several algorithms are reported for optimal load dispatch of a power system using constraints on emission, ramp-rat limit on generating stations and line flows [8, 9, 10].

The continuing growth of competition in American electricity markets is a consequence of the 178 passage of the Public Utility Regulatory Policies Act (PURPA). It established right of co-generation and helped independent power producers (IPPs) to sell electricity to local regulated investor-owned utilities (IOUs). This introduces several reforms in power sectors leading to a new regulatory environment. This allows IPPs to explore the market opportunity in newly emerging regulatory environment [11]. Because of introduction to open access in electricity market, IPPs having no transmission line(s) of their own are also granted the right to use the transmission system under transmission open access (TOA) arrangement. Two important issues are to be addressed while implementing TOA [12, 13]. They are economic and operational issues. The pricing of transmission services plays a crucial role in the success of deregulation since it determines whether the provided transmission services are economically beneficial to both the wheelers and customers. The usage of the transmission system need to be allocated in some manner to the participants in the electricity market [14] The revenues collected by IPPs for the transmission services need to pay for the transmission system [15].

This paper provides an algorithm for assigning transmission cost on both the wheelers (generators) and customers (loads). For this purpose, line power flow is represented as function of injections at the buses using sensitivity factors (SF).This allows solution of co-ordination equation for determining optimum generation scheduling involving transmission cost pertaining to the generating station. This ensures better co-ordination of transmission cost in optimal generation scheduling. Further, it allows assigning transmission cost to the loads that are drawing power through the line. The proposed algorithm is verified for a sample IEEE 30 bus system and results are presented.

II. LINE POWER FLOW IN TERMS OF BUS INJECTIONS

The expression for real power flow through a line connected between kth and mth buses is given as:

$$P_{km} = V_k^2 G_k + V_k V_m [G_{km} Cos\ \delta_{km} + B_{km} Sin\ \delta_{km}] \quad (1)$$

Where, $G_k = \frac{r_{km}}{r_{km}^2 + x_{km}^2}$ and $G_{km} = -G_k$ $B_{km} = \frac{x_{km}}{r_{km}^2 + x_{km}^2}$.

Change in power flow in a line is influenced by the change in voltage phase angles at both ends. Therefore, the change in line power flow between kth and mth buses with respect to change in voltage phase angles can be written as given below:

$$\Delta Pkm = \frac{P_{km}}{\partial \delta_k} \Delta\delta_k + \frac{P_{km}}{\partial \delta_m} \Delta\delta_m \quad (2)$$

The partial derivatives of Pkm with respect to δ can be written as follows:

$$\frac{P_{km}}{\partial \delta_k} = V_k V_m [-G_{km} Sin\ \delta_{km} + B_{km} Cos\ \delta_{km}]$$

$$\frac{P_{km}}{\partial \delta_m} = V_k V_m [G_{km} Sin\ \delta_{km} - B_{km} Cos\ \delta_{km}]$$

The change in bus voltage angle is primarily influenced by the change in real power injection at the buses [16]. Without loss of generality, taking bus number 1 as the slack bus, the change in bus voltage angle at kth bus in terms of bus injections can be expressed as [17]:

$$\Delta\delta_k = f_1^k \Delta P_1 + \sum_{j=2}^{N} f_j^k \Delta P_j \quad (3)$$

Where;

$$\gamma_i = \sum_{j=2}^{N} H_{1j} X_{ji} \quad \text{for} \quad i = 2, \cdots, N$$

$$\alpha_i = H_{ki} \quad \text{for} \quad i = 2 \cdots N$$

$$f_1^k = \frac{1}{\gamma_k \alpha_k}$$

$$f_j^k = -[\frac{\gamma_j}{\gamma_k \alpha_k} + \frac{1}{\alpha_k} \sum_{\substack{i=2 \\ i \neq k}}^{N} \alpha_i X_{ij}] \quad \text{for;} \quad \begin{matrix} j \neq 1 \\ j \neq k \end{matrix}$$

$$f_k^k = -\frac{1}{\alpha_k} \sum_{\substack{i=2 \\ i \neq k}}^{N} \alpha_i X_{ik}$$

The elements of matrix [H] are expressed as:

$$H_{ij} = V_i V_j (G_{ij} Sin\delta_{ij} - B_{ij} Cos\delta_{ij}) \quad \text{for;} \quad i \neq j$$

$$H_{ii} = \sum_{\substack{j=1 \\ j \neq i}}^{N} V_i V_j (-G_{ij} Sin\delta_{ij} + B_{ij} Cos\delta_{ij})$$

The elements Xij are the element of inversion of matrix [H], i.e., $[X] = [H^{]-1}$. Similarly for another bus m, the change in voltage phase angle in terms of change in real power injections can be written as follows.

$$\Delta\delta_m = f_1^m \Delta P_1 + \sum_{j=2}^{N} f_j^m \Delta P_j \quad (4)$$

$$f_1^m = \frac{1}{\gamma_m \alpha_m}$$

$$f_j^m = -[\frac{\gamma_j}{\gamma_m \alpha_m} + \frac{1}{\alpha_m} \sum_{\substack{i=2 \\ i \neq m}}^{N} \alpha_i X_{ij}] \quad \text{for;} \quad \begin{matrix} j \neq 1 \\ j \neq m \end{matrix}$$

$$f_m^m = -\frac{1}{\alpha_m} \sum_{\substack{i=2 \\ i \neq m}}^{N} \alpha_i X_{im}$$

Substituting Δδk and _Δδm from equation (3) and (4) in equation (2) we have.

$$\begin{aligned} \Delta P_{km} &= \frac{\partial P_{km}}{\partial \delta_k} \sum_{j=1}^{N} f_j^k \Delta P_j + \frac{\partial P_{km}}{\partial \delta_m} \sum_{j=1}^{N} f_j^m \Delta P_j \\ &= \sum_{j=1}^{N} (\frac{\partial P_{km}}{\partial \delta_k} f_j^k + \frac{\partial P_{km}}{\partial \delta_m} f_j^m) \Delta P_j \\ &= \sum_{j=1}^{N} f_j \Delta P_j \end{aligned} \quad (5)$$

Above equation gives the change in power flow through a line connected between k th and m th buses in terms of sensitivity factors (SF) fi and change in power injections ΔP. If the change in injections are taken as Pj; for: j = 1...N then it should provide power flow for the line connected between kth and mth buses as follows.

$$P_{km}^{cal} = \sum_{j=1}^{N} f_j P_j = [SF][P] \qquad (6)$$

Sensitivity factors depend on the value of [δ] and [V] along with the network parameters of the system and their relation to line power flow is nonlinear. Therefore, computation of line power flow based on bus power injections using equation (6) may differ from actual line power flow (Pkm), as sensitivity factors fi are derived as constant value with respect to the operating condition of the system. Therefore, it is required to modify sensitivity factors fi to take care of the error in power flow between actual line flow(Pkm) and calculated power flow (P_{km}^{cal}) for the operating condition of the system. The error is distributed among the buses as described below.

$$P_{km}^{error} = P_{km} - P_{km}^{cal} = [SF][P^{error}] \qquad (7)$$

Where, P_i^{error} is the error in injections at ith bus due to error in the line of flow. Since [SF] is a row matrix, therefore, [P_i^{error}] values are to be calculated using pseudoinverse technique. Now, solving the above equation the value of P_i^{error} can be written as:

$$\left[P^{error}\right] = \left[SF\right]^T \left(\left[SF\right]\left[SF\right]^T\right)^{-1} P_{km}^{error} \qquad (8)$$

which, yields

$$P_i^{error} = \frac{f_i P_{km}^{error}}{\sum_{j=1}^{N_L+N_G} f_j^2} \quad \text{for} \quad i = 1..N_L + N_G \qquad (9)$$

The buses with no injections are excluded. The equation (7) is rearranged as:

$$\begin{aligned} P_{km} &= P_{km}^{cal} + P_{km}^{error} = \sum_{i=1}^{N_L+N_G} f_i(P_i + P_i^{error}) \\ &= \sum_{i=1}^{N_L+N_G} [f_i(1 + \frac{P_i^{error}}{P_i})]P_i \\ &= \sum_{i=1}^{N_L+N_G} pf_i P_i \qquad (10) \end{aligned}$$

Where, Pi will be the injection at ith bus for the current operating condition of the system.

III. COST FUNCTION FOR GENERATING STATIONS AND LOADS

Equation (10) allows us to determine contribution from different participating generating stations and loads in respect of line flow of the selected line. If tckm is the cost of transmission for jth transmission line between kth and mth buses, assigned to the generating stations based on its participation in the line flow in Rs/Mw, then, the transmission cost on generations for the line will be:

$$TCG_j = \sum_{k=1}^{N_G} tc_j pf_k P_{Gk} \qquad (11)$$

Similarly, the cost of transmission, which is to be contributed by the consumer, is as follows:

$$TCL_j = \sum_{k=1}^{N_L} tc_j pf_k(-P_{Dk}) \qquad (12)$$

The positive pfk values are to be used for determining transmission cost for generating stations, whereas, the negative pfk values are to be used for determining transmission cost for loads connected to the load buses. If several lines (NT) are to be included in the process, then total cost of transmission assigned to generating stations and loads will be given by the equations (13) and (14).

$$TCG = \sum_{i=1}^{NT} TCG_i = \sum_{i=1}^{NT}\sum_{j=1}^{N_G} tc_i pf_{ij} P_{Gj} \qquad (13)$$

And

$$TCL = \sum_{i=1}^{NT} TCL_i = \sum_{i=1}^{NT}\sum_{j=1}^{N_L} tc_i pf_{ij}(-P_{Dj}) \qquad (14)$$

IV. OPTIMUM GENERATION SCHEDULING

The Lagrange multiplier method is used for solution of the optimum scheduling using co-ordination equation for allocation of generations from different generating units. The co-ordination equation is as follows:

$$\frac{\partial F_T}{\partial P_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda$$

The cost function for ith generating station, including cost of transmission can be expressed as:

$$F_T = \sum_{i=1}^{N_G} a_i P_{Gi}^2 + b_i P_{Gi} + [\sum_{j=1}^{N_T} tc_j pf_{ji} P_{Gi}] + c_i \quad (15)$$

The change in system loss can be expressed as[7]:

$$\Delta P_L = \sum_{i=2}^{N} (1 + \gamma_i)[\Delta P_{Gi} - \Delta P_{Di}] \qquad (16)$$

Therefore,

$$P_{Gi} = \frac{-[\gamma_i \lambda + b_i + \sum_{j=1}^{NT} tc_j pf_{ji}]}{2a_i} \quad \text{for;} \ i = 1.N_G \quad (17)$$

Equation (17) is solved by an iterative procedure to arrive at the condition: |PGT − (PDT + PL)| ≤ ε

V. SOLUTION PROCEDURE:

Optimum load dispatch in a power system is solved through an iterative procedure involving the solution of co-ordination equations and followed by a load flow analysis until the balance among generation, load demand and system loss is satisfied. The solution steps for the optimal load dispatch problem are as follows:

1. Carry out a load flow analysis to determine base case operating condition of the system. Compute PDT and P_L^o and set K=0.

2. Compute Pkm for the line, if Pkm < 0; then exchange k and m to make Pkm > 0 for all lines considered for assigning transmission cost.

3. λmax = MAX{2aiP2Gi + biPGi} and λmin = MIN{bi}, where λmax = Maximum possible value of λ and λmin = Minimum possible value of λ.

4. K = K + 1.

5. Determine SF using equation (10).

6. λa = λmax and λb = λmin.

7. λ = $\frac{\lambda_a+\lambda_b}{2}$.

8. i = 1.

9. Compute PGi using equation (17).

10. i = i + 1.

11. if i ≤ NG go to step - 9.

12. Determine PGT =$\sum_{i=1}^{NG} P_{Gi}$.

13. if (PGT − PDT − P_L^{K-1}) > 0.0 set λa = λ.

14. if (PGT − PDT −P_L^{K-1}) < 0.0 set λb = λ.

15. if (PGT − PDT −P_L^{K-1}) > ε go to step - 7.

16. Conduct load flow and determine P_L^K.

17. if $|P_L^K - P_L^{K-1}|$ > ε go to step - 3.

18. stop.

VI. SIMULATION AND RESULTS

IEEE 30 bus system is adopted for verification of the algorithm proposed in the paper. Simulations were carried out for different load power factors keeping loads at the buses same as base loads. The base loads at the buses in MWs are as follows:

P_{D2}= 13.91	P_{D3}= 29.12	P_{D4}= 22.88
P_{D5}= 19.76	P_{D7}= 42.64	P_{D8}= 39.0
P_{D10}= 11.44	P_{D12}= 14.56	P_{D14}= 8.06
P_{D15}= 10.66	P_{D16}= 4.55	P_{D17}= 11.70
P_{D18}= 4.16	P_{D19}= 25.35	P_{D20}= 2.86
P_{D21}= 22.75	P_{D23}= 1.30	P_{D24}= 3.51
P_{D26}= 4.55	P_{D29}= 4.42	P_{D30}= 13.78

During simulations, different numbers of lines were considered for assigning transmission cost to the generating stations and loads. Four transmission lines were considered for assigning transmission cost on generating stations and loads with combinations of three and four at a time. The transmission cost for all lines are taken as 1.8 Rs/MW. The transmission lines considered are as follows: line no. 1 between bus number 5 and 7, line no. 2 between bus number 9 and 10, line no. 3 between bus number 12 and 15, line no. 4 between bus number 10 and 20. The coefficients of cost functions for generating stations and their limits on generations are provided in Table 1.

The optimal load dispatch results obtained using the proposed algorithm were compared with optimal load dispatch results obtained using the method described in reference[7] without incorporating trans- mission cost in the cost functions, and SFs described in the paper are used for determining transmission cost and cost on loads. Case-I: first three lines are considered for assigning transmission cost on generating stations and loads with load power factors Lpf 0.8, 0.85 and 0.9. Optimal load dispatch results in terms of transmission cost, generating cost and cost on loads are presented in table 2.

Table-1: Co-efficients of cost function and generation limits for the generating stations for the IEEE 30 bus system.

Gen. No.	a_i $\frac{Rs}{Mw^2}$	b_i $\frac{Rs}{Mw}$	c_i Rs	PG_{maxi} Mw	PG_{mani} Mw
G-1	0.004	3.3	10.00	120.0	20.0
G-2	0.005	3.3	10.00	110.0	20.0
G-3	0.0035	3.3	10.00	110.0	20.0
G-4	0.009	3.3	10.00	60.0	20.0
G-5	0.009	3.3	10.00	60.0	20.0
G-6	0.009	3.3	10.00	60.0	20.0

Table-2: Optimal load dispatch results for Case-I with load power factors - L_{pf} 0.8, 0.85 and 0.9 respectively

Based on the proposed Algorithm			Based on the Reference[7]		
F_T	TCG	TCL	F_T	TCG	TCL
1318.5	110.0	347.2	1325.4	123.4	350.8
1309.6	106.0	350.2	1316.6	119.3	353.7
1301.9	102.3	353.2	1309.2	115.6	356.7

Case-II: All four lines are considered for assigning transmission cost on generating stations and loads with load power factor 0.8, 0.85 and 0.9. Optimal load dispatch results in terms of transmission cost, generating cost and cost on loads are presented in Table 3.

Table-3: Optimal load dispatch results for Case-II with load power factors - L_{pf} 0.8, 0.85 and 0.9 respectively

Based on the proposed Algorithm			Based on the Reference[7]		
F_T	TCG	TCL	F_T	TCG	TCL
1418.7	210.9	371.2	1424.7	222.7	375.4
1407.3	204.5	374.6	1413.5	216.2	378.8
1397.3	198.6	378.3	1403.8	210.2.8	382.4

It is very important to see the difference between P_{km} and P_{km}^{cal}, ie P_{km}^{error} during the course of iterations of optimal load dispatch analysis. Therefore, P_{km}^{error} values were recorded for all four lines during optimal load dispatch iterations with load power factors 0.8, 0.85 and 0.9. It was observed that for all load power factors the variations are quite close. Therefore, the variation of P_{km}^{error} in term of % change $(=\frac{P_{km}^{error}}{P_{km}}100\ \%)$ for optimal load dispatch iterations with load power factor 0.8 are presented in the graph shown in Figure - 1.

The simulation results for IEEE 30 bus system shows that incorporation of cost coefficients in generating stations due to power flow contribution by generating stations (through lines considered for assigning transmission cost on generations and loads) ensures better co-ordination during optimal load dispatch analysis. This results in reduction of generation cost (F_T), cost pertaining to transmission cost assigned to generating stations (TCG) and to loads (TCL). Again, Figure-1 shows that % change in P_{km}^{error} has not exceeded even 5% of their operating power flow. As such, corrections on fi to obtain pfi are much more smaller.

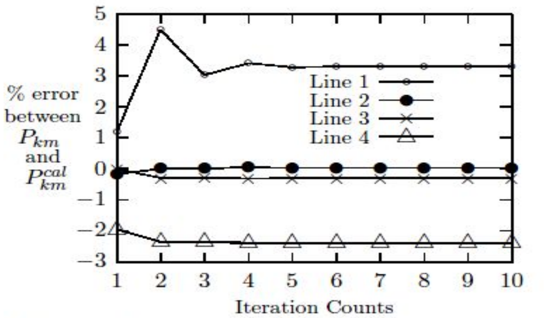

Figure-1. % change in P_{km}^{error} during optimal load dispatch iterations.

VII. CONCLUSION

The proposed algorithm for assigning transmission cost on participating generating stations and loads in terms of sensitivity factors provide a basis for the power system planner and operator to enforce revenue on power transmission through transmission lines. It will be an useful tool for determination of revenue on power flow through transmission lines by the transmission service provider in the face of emerging open access in electricity market. The simulation and analysis carried out for the sample 30 bus system indicates that the proposed algorithm for optimal load dispatch ensures reduction of generation cost (F_T), cost pertaining to transmission cost assigned to generating stations (TCG) and to loads (TCL). The correction on fi to obtain pfi are quite small as % change in P_{km}^{error} has not exceeded even 5%. Finally, the formulation for sensitivity factors provides facility for assigning transmission cost to the slack bus also.

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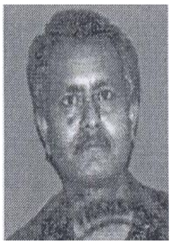

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