Study of Graded Index and Truncated Apertures Using Speckle Images

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Abstract- In this paper, apertures in the form of graded index distribution and truncated apertures are investigated. The 1st type of graded index aperture has central black zone followed by steps with increased intensity directed outwards. The second aperture has successive black and transparent annuls with central opaque zone. The third has four different truncated apertures. Digital diffuser is fabricated to get modulated speckle images for all described apertures. The profile of the speckle images and the corresponding autocorrelation profiles are plotted and the speckle sizes are computed for the different cited apertures. A Mat-lab code is written to compute and plot all of the above images.

Keywords- Graded Index Apertures; Speckle Images; Numerical Image Processing

I. INTRODUCTION

Recently, speckle formation using diffusers modulated by linear, conical, quadratic, obstructed circular, deformed kidney and elliptical modulated apertures [1-4] is investigated. We showed, using Fourier optics analysis, that the speckle features for the above mentioned apertures are dependent upon the aperture distribution. Others proposed a multiple exposure specklegram by using an optical system whose multiple aperture pupil changes between exposures [5]. The characteristics of speckle patterns generated through multi-aperture pupils are theoretically analyzed also based on Fourier techniques [6]. In another recent work they analyzed speckle images generated when a diffuser illuminated by coherent light is image by a lens having a pupil mask with multiple apertures forming a closed curve to obtain cluster speckle structure [7]. The cluster structure results from the complex modulation produced inside each speckle which is generated by multiple interference of light through the apertures. In particular, when the apertures are uniformly distributed along a closed curve, the resulting image speckle cluster replicates the pupil aperture distribution. The authors in [7] showed from the experimental and theoretical simulations that the cluster features are dependent on the apertures distribution and the size of the closed curve.

The speckle is considered as an important topic for optical imaging of biomedical objects with irregular shapes such as tumor and human skin. The average speckle size of a speckle image can be estimated by calculating the auto-covariance function of the digitized intensity speckle pattern. The auto-covariance function corresponds to the normalized autocorrelation function of the intensity which has zero base and its full width at half maximum (FWHM) provides a reasonable measurement of the average width of speckle [8-10].

In the present work, modulated graded index and truncated apertures are considered. The speckle images of diffusers using these modulated apertures are examined. Hence, the autocorrelation function of the modulated speckle images is computed and plotted. The average speckle size for these modulated speckle images is computed from the autocorrelation of speckle image.

II. THEORETICAL ANALYSIS

A numerical aperture of definite number of steps (N) of central black zone and increases gradually outwards, as shown in the Fig.1, is investigated. This aperture is considered as a graded index aperture.

This graded index aperture is represented mathematically as follows:

\[ P_{\text{graded}}(x,y) = p_0 + \alpha \sum_{i=2}^{N} p_i(x,y) \]  

(1)

\( p_0 \) is the central black zone of i=1, \( \alpha \) is a weighting factor which has the values of 0.2, 0.3,...,1 and N is the total number of graded steps.

The 2nd aperture assumes successive black and transparent annuli of central black zone and a computerized fabrication of this aperture is represented as in same Fig. 1. This aperture is similar to the first aperture but with equal steps of a weighting factor \( \alpha = 1 \).

This aperture is similar to the graded index aperture with \( \alpha = 1 \) and is represented mathematically as follows:
$P_{hyw}(x, y) = p_0 + \sum_{i=2}^{N} p_i \cos(\theta)$

The 3$^{rd}$ truncated aperture is represented as follows:

$P_{truncated}(x, y) = \cos(\rho \theta) = 1 \quad ; \quad |\rho| \leq 1$

$\rho = (x^2 + y^2)^{1/2}$. The letter $\rho$ is the radial coordinate in the aperture plane.

Fig. 1  The first aperture from the left has a graded index aperture, in the middle is black and transparent concentric annular aperture, and in the right is a circular aperture. All apertures are of dimension 1024x1024 pixels and radius $R = 128$ pixels.

Four different sections from the circular aperture are chosen as follows:

The 1$^{st}$ is a quarter of circle in the range $0 \leq \theta \leq \pi/2$; the 2$^{nd}$ is a half of circle and its azimuthal range is $0 \leq \theta \leq \pi$; the 3$^{rd}$ is three quarters of circle and its azimuthal range is $0 \leq \theta \leq 3\pi/2$; and the 4$^{th}$ has the two conjugate quarters or fan structure in the azimuthal range $0 \leq \theta \leq \pi/2$, and $\pi \leq \theta \leq 3\pi/2$.

A diffuser $d(x, y)$ considered as a randomly distributed object is used to fabricate a speckle pattern in case of the graded index apertures for a variable weighting factor $\alpha$ represented in Equation (1) and in the case of step index aperture of a weighting factor $\alpha = 1$ known as black and transparent annuli Equation (2), and the truncated apertures Equation (3).

In case of coherent uniform illumination, the transmitted complex amplitude becomes:

$A(x, y) = p(x, y) \cdot d(x, y)$

Where $p(x, y) = p_{graded}(x, y)$ ; for graded index aperture, $p(x, y) = p_{hyw}(x, y)$ ; for B/W step index aperture, and $p(x, y) = p_{truncated}(x, y)$ ; for truncated apertures.

The speckle pattern is obtained in the focal plane of the imaging lens, using coherent laser illumination, by operating the Fourier transform upon Equation (4) to get:

$B(u, v) = F.T.\{A(x, y)\} = F.T.\{p(x, y)\cdot d(x, y)\} = h(u, v) * D(u, v)$

The symbol (*) is used for convolution operation, $h(u, v) = F.T.\{p(x, y)\}$ and $D(u, v) = F.T.\{d(x, y)\}$ is the conventional speckle image in case of uniform circular aperture where $p(x, y) = 1$ inside the aperture and is zero outside the aperture.

### III. COMPUTATION OF THE AUTOCORRELATION OF THE MODULATED SPECKLES

The modulated speckles are constructed from the operation of the Fourier transform of the multiplication of the diffuser and the modulating aperture, represented in Equation (5). The autocorrelation of the speckle image is obtained using Mat-lab code.

The autocorrelation function of Equation (5) is written as:

$c(u, v) = B(u, v) \otimes B(u, v)$

The symbol $\otimes$ is used for the autocorrelation operation of the two similar speckle images.

Assuming that the complex amplitude of the modulated speckle image given by Equation (5) is rewritten as follows:
\[ D_{\text{modulating}} = h(u,v) \otimes D(u,v) \quad \text{where } B(u,v) = D_{\text{modulating}} \quad (7) \]

Hence, the autocorrelation product (6) is operated on two symmetric modulated apertures as shown in Equation (8).

\[ c(u,v) = D_{\text{modulating}} \otimes D_{\text{modulating}} \quad (8) \]

IV. RESULTS AND DISCUSSION

Three different apertures are numerically constructed as shown in Fig. 1. The 1st aperture from the left has a graded index aperture, in the middle is black and transparent concentric annular aperture, and in the right is a circular aperture. All plotted apertures are of dimensions 1024x1024 pixels and radius R = 128 pixels. Another different set of truncated apertures are plotted in Fig. 2. From the left, the 1st has three quarters of a circular aperture, the 2nd is a half circular aperture, the 3rd has only one quarter, while the 4th has two symmetric quarters in the form of a fan aperture.

Fig. 2 Four different truncated apertures, where a) three-quarters aperture, b) semi-circular aperture, c) one quarter of circular aperture , d) two conjugate quarters in the form of fan aperture. All apertures are of dimension 1024x1024 pixels

The speckle images for diffusers modulated by the described apertures shown in Figs. 1 and 2 are obtained by operating the Fourier transform of the multiplication product of the diffuser and the aperture. Hence, the modulated speckle images shown in Figs. 3 and 4 are considered as the convolution product of the ordinary speckle image and the point spread function (PSF) of the defined aperture. It is shown that the three speckle images shown in Fig. 3 are completely different since the PSF is different for each aperture. Also, the speckle images shown in Fig. 4a-d are dependent upon PSF of the truncated apertures. All speckle images are of dimensions 256x256 pixels.

Fig. 3 From the left, speckle images are shown corresponding to the diffuser provided with the apertures shown in the Fig. (1). Speckle image for the graded index aperture, b) speckle image for the concentric B/W annuli, and c) speckle image for the ordinary uniform circular aperture
Fig. 4a Speckle pattern for quarter of circular aperture of dimensions 256x256 pixels

Fig. 4b Speckle pattern for half circular aperture

Fig. 4c Speckle pattern for three quarter circular aperture

Fig. 4d Speckle pattern for a fan aperture
The five profile line shapes of the different three speckle images with the arrangement shown in Fig. 3 at lines 50, 110, 128, 160, 190 pixels are shown in Fig. 5a-e. For each graph three curves are plotted, the upper curve for the speckle corresponding to graded index aperture, in the middle the curve is for step index B/W aperture, and the lower curve is for a circular uniform aperture. The three profiles for each of the five graphs are completely different since the apertures are different which is in a good agreement with graphs shown in Fig. 3. This is attributed to the convolution product of the ordinary speckle in case of diffuser and the PSF of the different apertures.
Fig. 5a-e: The five profile line shapes of the different three speckle images with the arrangement shown in Fig. 3 at lines 50,110,128,160,190 pixels are shown in Fig. 5a-e. In all five plots (a–e), three plots are shown. The upper is the plot for the speckle using graded index aperture, in the middle is the plot for the speckle using the concentric B/W annuli, and the lower is the plot in case of speckle using the circular aperture.

A separate profile line shapes of the speckle images corresponding to the diffuser provided with the graded index aperture are shown in Fig. 6a. While the profile line shapes of the speckle images using the step index black and white concentric annular aperture are shown in Fig. 6b. The comparative line shapes of the speckle images corresponding to the diffuser provided with the circular uniform aperture are shown in Fig. 6c. It is noted that all the curves corresponding to the three separate graphs starting from the upper are plotted at lines 50,100,150, and 200 pixels respectively.
Fig. 6a: The plots, from the upper are taken at lines 50, 100, 150, and 200 pixels respectively.

Fig. 6b: The plots, from the upper are taken at lines 50, 100, 150, and 200 pixels respectively.

Fig. 6c: The plots, from the upper are taken at lines 50, 100, 150, and 200 pixels respectively.

Fig. 6: A separate profile line shapes of the speckle images corresponding to the diffuser provided with the graded index aperture are shown in Fig. 6a. While the profile line shapes of the speckle images using the step index black and white concentric annular aperture are shown in Fig. 6b. The comparative line shapes of the speckle images corresponding to the diffuser provided with the circular uniform aperture are shown in Fig. 6c. It is noted that all the curves corresponding to the three separate graphs starting from the upper are plotted at lines 50, 100, 150, and 200 pixels respectively.
A uniform circular aperture of a radius=64 pixels on a matrix of dimensions 2048x2048 pixels is plotted on the left while on the right is the speckle pattern of the diffuser provided with this aperture and has dimensions of 256x256 pixels as shown in Fig. 7. The field of view is 4.5 mm x 3.6 mm for the speckle image.

![Fig. 7 A small circular aperture of radius =64 pixels and the corresponding speckle image](image)

The autocorrelation function corresponding to the above speckle image obtained from the same diffuser provided with the circular aperture of radius 64 pixels is plotted as in Fig. 8. On the left is the peak along the x-axis while on the right is the peak for the y-axis summation.

![Fig. 8 Profiles of the autocorrelation of speckle using circular aperture](image)

The FWHM of the speckle size corresponding to the 20 pixels shown on the autocorrelation peak, along each of the x-axis and y-axis, is equal to: 

$$\sigma_x = (1/2) \times (4.5 \text{ mm} / 512 \text{ pixels}) \times (20 \text{ pixels}) = 88 \mu\text{m}$$

While the FWHM of the speckle size corresponding to the 20 pixels shown on the autocorrelation peak along the y-axis is equal to: 

$$\sigma_y = (1/2) \times (3.6 \text{ mm} / 512 \text{ pixels}) \times (20 \text{ pixels}) = 70 \mu\text{m}$$

The autocorrelation images of the truncated apertures are plotted as in Fig. 9.
V. CONCLUSION

It is shown that the speckle images obtained for diffusers provided with graded index and truncated apertures are different from those obtained with uniform circular aperture. The autocorrelation of the speckle images is plotted and the FWHM of the speckle images is computed. The autocorrelation images of the different apertures showed discrimination between the

Fig. 9 Autocorrelation images of the truncated apertures, the upper for the quarter, the next for half of circle the 3rd for three quarters of circle, and the last image for the fan aperture
apertures. Also, the speckle images obtained in case of truncated apertures are obtained and showed a noticeable difference. The agglomeration of speckle patches and its direction is dependent upon the shape of the considered apertures.

It is concluded that the different speckle images are basically dependent upon the point spread function of the examined aperture. Consequently, the modulated speckle image is the convolution product of the ordinary speckle image obtained in case of uniform circular aperture and the point spread function of the modulating aperture.

The application of the simulated results in testing the microscopic apertures is proposed by the investigation of the different speckle images and the correlation images of apertures. Also, it is possible to do experimental validation of the proposed speckle technique using modulated microscopic objectives. For example, in order to compute the experimental point spread function (PSF), it is necessary to construct sophisticated arrangement of spatial filtering using the modulated apertures and pinholes to obstruct the legs of the diffraction pattern located in the focal plane of the objectives.

REFERENCES


The author was born in Cairo, Egypt at 13/05/1951 and is graduated From the Faculty of science, Ain Shams University. B.Sc. excellent with honors degree in June 1973 and M.Sc. in Laser Optics in December 1976. Hence I have got a scholarship from the France in 1978 and I have obtained the title Doctor of Physics in July 1985 from the University of Paris 11) in Laser scanning microscope. Now I have got the title of Prof. of Theoretical Optics and Laser in 1998 till now as a Prof. Emeritus at Physics Department, Faculty of Science, Ain Shams University. My research contribution exceeds 45 international publications and a book on Poly-chromatic image processing (Laser Applications). I have reviewed manuscripts in international reviews like Optics and Laser Technology, Optics and Laser in Engineering, Optical Engineering, and Egyptian Journal of Solids. I’m a member of New York Academy of Science (NYAS) in 1997,1998 and American Association for the Advancement of Science (AAAS) in 1998.