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Analysis of Belting Stiffness Transmission Impact on Rotating Mass Motion Law of Technological Machines

D.A. Mamatova\textsuperscript{1}, Shangyong Zhang\textsuperscript{2}, A. Djuraev\textsuperscript{3}, M.A. Mansurova\textsuperscript{4}

\textsuperscript{1,3,4}Theoretical and Applied Mechanics department, Tashkent Institute of Textile and Light Industry, 5 Shohjohon str., Yakkasaray District, Tashkent, 100100, Uzbekistan

\textsuperscript{2}School of Textile Science and Engineering, Wuhan Textile University, No. 1 Yangguang Road, Jiangxia District, Wuhan, 430200, China

\textsuperscript{2}shangyong.zhang@wtu.edu.cn

Abstract-The aims of this research are to develop a new scheme of belt transmission with an integral driven pulley and tensioning roller as well as to determine the laws of motion for the main and bottom shafts of the sewing machine as they depend on the rigidity of the elastic elements of the driven belt pulley with justification of settings. The article describes a new diagram of a belt drive with a split driven pulley and eccentric tension roller. The laws of the driver masses' motion of the sewing machine were obtained and analyzed.

Keywords- Belt Drive; Split Pulley; Torsion; Sewing Machine; Rigidity; Torque

I. INTRODUCTION

Sewing-machines have various destinations and are produced in a number of classes. The majority of sewing-machines have a drive in which the rotary motion from the electric motor is transferred by a belt drive, and motion from the main shaft to the bottom shaft is transferred by a toothed belt drive (Fig. 1) with the transfer relation equal to 1. Such mechanisms as the slider, thread take up, and winder receive their motion from the main shaft. The shuttle and transfer mechanism obtain their motion through the bottom shaft. The torque movement both on the main and bottom shafts have a complicated character due to the variability of impacts from the corresponding machines’ mechanisms. The problem occurs especially during processing of thick fabrics. The driven belt usually experiences great shocks at stitching of clothing with high densities. Different types of belts with various values of stiffness as well as the drive of the main shaft, Fig. 1, have been designed by machine building plants in recent years.

Fig. 1 General view of the sewing machine drive, 1-motor, 2-pulley, 3-belt, 4-driven composite pulley, 5-tension pulley, 6-main shaft, 7-drive pulley, 8-driven composite pulley, 9-bottom shaft, 10-toothed belt

It is known that the driving belts transfer torsion torque in the drives of most technological machines and some sewing machines [1]. The belt transmission evens the fluctuations of the rotational movement and the angular velocity during the running process. Variation of speed and torsion torque on the shaft of the driven pulley will increase at high fluctuations of technological loads [2]. To reduce the amplitude of these oscillations, we recommend a new design of a belt transmission [3]. Fluctuations in the rotational moments, $M_i$, and angular velocities, $\omega_i$, of the working shafts arise because of the changeable technological loads, as well as the variability of the inertial forces. This may lead to stitching slips and malfunctions in looping together with increased thread breakage. Therefore, we recommend using our design in the drive of the sewing machine.

II. ANALYTICAL METHOD

A method of mass reduction was adopted to design the scheme, and Lagrange’s equations of the second order were used to
analyse the system of differential equations.

In the series of technological machines operating under large speed modes, sewing machines in particular, the load variations’ shock absorption by the drive belt is not sufficient. We recommend belt drives, with split driven pulleys, with springing elements, which allows considerable decreasing of the load variations’ peak values of the main and bottom shafts and sewing machine’s electric drive shaft (Fig. 2). The drive shaft (1), through the belt (3), imparts the rotation to the driven pulley (2), which results in rotation of the tension roller (4). The rotational motion from the rim (5) through the ring-type spring bush (7) is imparted to the hub (6), which is rigidly connected to the driven pulley’s shaft (2). The driven pulley’s shaft (2) is connected to the main shaft of the sewing machine (not shown on the figure).

![Fig. 2 Belt drive with split driven pulley and spring tension roller](image)

At the performance of the technological process on the sewing machine by the main shaft, the load on the driven pulley’s shaft (2) is changed. These variations in the moment of resistance are transmitted to the rim (5) through the ring-type spring bush (7) and, further, through the drive pulley (1) and to the electric drive. Thus, the ring-type spring bush (7) absorbs the peak values of the moment of resistance (load). By choosing the necessary spring-dissipative properties (rubber material), it is possible to manage the extent of absorption in the peak load values. Thus, the rotation of the rim (5) of the driven pulley (2) is sufficiently smoothed out. Consider the scheme on Fig. 3, which uses the recommended belt drive.

![Fig. 3 The dynamic model of the sewing machine’s drive. I – the mass 1 of the electric motor’s rotor with the drive pulley 2, II – the pulley rim’s mass 4, III – the mass of pulley hub 4 with the main shaft 6 with masses of segments of the needle mechanism and take up and drive pulley 7, IV – the mass of the driven pulley’s rim 8, V – the mass of the pulley hub 8 with the bottom shaft and segments in the relevant sewing machine’s mechanisms](image)

At patterning of the dynamic and mathematical motion models of the sewing machine’s mechanisms, the given inertia moments of the main and bottom shaft were taken as constant, though they can vary in small values on the segments of sewing machine’s toggle mechanisms [4]. The tension roller with a springing element was used in the belt drive, from the electric motor to the main shaft. Such design allows, to some degree, a decrease of the oscillations of torques and speeds on the motor’s shaft. At the same time, taking into account the tension roller’s springing element of the belt drive and hanging loads, the transmission ratio of the mass is changed.

The ratio, $U_{1,2}$, between the angular speeds of the sewing machine’s drive masses, $\dot{\phi}_1$, will be:

$$U_{1,2} = \frac{\dot{\phi}_1}{\dot{\phi}_2} ; \ U_{2,3} = \frac{\dot{\phi}_2}{\dot{\phi}_3} = 1.0 ; \ U_{3,4} = \frac{\dot{\phi}_3}{\dot{\phi}_4} = 1.0 ; \ U_{4,5} = \frac{\dot{\phi}_4}{\dot{\phi}_5} = 1.0$$

(1)

From the given equations in (1) is seen that only $U_{1,2}$ has a variable value. The other transmission ratios, to some degree, have their mean values equal to one. The electric motor, in the mathematical model of the drive, was taken into account as the dynamic mechanical characteristic according to [3, 5].

Applying the Lagrange equations [6, 7] the equations of motion for the sewing machine’s drive were obtained with consideration of the torque moment, $M_m$, critical torque moment, $M_c$, on the motor, critical sliding value, $S_c$, and critical angular velocity, $\omega_c$:

$$\frac{dM_m}{dt} = 2M_c\omega_c \left( \omega_0 - \frac{d\phi_5}{dt} \right) - \omega_0 S_c M_m$$

(2)
Moment of inertia for the first mechanism of the sewing machine considers the moment of inertia, \( I_{1\text{rot}} \), of the rotor and the moment of inertia, \( I_{1\text{pul}} \), of the drive pulley, as well as their rigidity, \( C_1 \), dissipation of the belt, \( d \), and the change in the angle, \( \gamma_{1,2} \), of their rotation.

\[
\left( I_{1\text{rot}} + I_{1\text{pul}} \right) \frac{d^2 \phi_1}{dt^2} = M_m - C_1 \left[ \phi_0 - \left( \frac{r_1 \cos(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \phi_2 \right]
\]

\[
\left[ 1 - \phi_2 \left( \frac{r_1 \sin(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \right] \left( \frac{r_1 \cos(\arctan \gamma_2)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_2 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \phi_2
\]

\[
\left( \frac{\Delta r \cos^2 \phi_1/\phi_1 + r_1 \cos 2\phi_1/\phi_1}{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2}} \right) - \frac{d \phi_1}{dt} - \left( \frac{r_1 \cos(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \frac{d \phi_2}{dt}
\]

(3)

The moment of inertia, \( J_{2,0} \), for the rim of the driven pulley of the sewing machine is described by the following equation.

\[
J_{2,0} \frac{d^2 \phi_2}{dt^2} = C_1 \left[ \phi_1 - \left( \frac{r_1 \cos(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \phi_2 \right]
\]

\[
\left[ \frac{r_1 \cos(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right] \left( \frac{r_1 \sin(\arctan \gamma_2)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_2 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \phi_2
\]

\[
+ \frac{d \phi_1}{dt} - \left( \frac{r_1 \cos(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right) \frac{d \phi_2}{dt}
\]

\[
\left( \frac{r_1 \cos(\arctan \gamma_1)}{r_1 \cos(\arctan \Delta_2)} + \frac{\sqrt{r_1^2 \cos^2 \gamma_1 + \Delta_2^2} - \frac{r_2}{r_1}} \right) = C_1 (\phi_1 - U_{2,3}) - d_2 \left( \frac{d \phi_2}{dt} - U_{2,3} \frac{d \phi_2}{dt} \right)
\]

(4)

The moment of inertia for the third mechanism of the sewing machine takes into account the moment of inertia, \( J_{3\text{m}} \), of the needle mechanism, the moment of inertia, \( J_{3\text{m}} \), of the main shaft, the moment of inertia, \( J_{3\text{h}} \), of the driven pulley’s hub, the moment of inertia, \( J_{3\text{r}} \), of the thread take up mechanism, and the moment of inertia, \( J_{3\text{dr}} \), of the drive shaft of the toothed belt drive.

\[
\left( J_{3\text{m}} + J_{3h} + J_{3r} + J_{3dr} \right) \frac{d^2 \phi_3}{dt^2} = U_{2,3} C_1 (\phi_1 - U_{2,3}) + U_{2,3} d_2 \left( \frac{d \phi_2}{dt} - U_{2,3} \frac{d \phi_2}{dt} \right)
\]

\[
- C_4 (\phi_3 - U_{3,4}) - d_4 \left( \frac{d \phi_4}{dt} - U_{3,4} \frac{d \phi_4}{dt} \right) - M_n,
\]

(5)

The moment of inertia, \( J_{4,0} \), of the driven pulley’s rim in the toothed belt drive considers its rigidity, \( C_4 \).

\[
J_{4,0} \frac{d^2 \phi_4}{dt^2} = U_{3,4} C_3 (\phi_3 - U_{3,4}) + U_{3,4} d_3 \left( \frac{d \phi_4}{dt} - U_{3,4} \frac{d \phi_4}{dt} \right) - C_4 (\phi_4 - U_{4,5}) - d_4 \left( \frac{d \phi_4}{dt} - U_{4,5} \frac{d \phi_4}{dt} \right)
\]

(6)
The moment of inertia for the fifth mechanism of the sewing machine takes into account the moment of inertia, \( J_{5bh} \), of the shuttle mechanism, the moment of inertia, \( J_{5pb} \), of the push bar mechanism, the moment of inertia, \( J_{5h} \), of the driven pulley’s hub in the toothed belt drive, the moment of inertia, \( J_{5sh} \), of the bottom shaft, and the moment of inertia, \( J_{5tm} \), of the transfer mechanism.

\[
\left( J_{5bh} + J_{5pb} + J_{5h} + J_{5sh} + J_{5tm} \right) \frac{d^2 \phi_5}{dt^2} = U_{4,5} C_4 \left( \phi_2 - U_{4,5} \phi_3 \right) + U_{4,5} d_4 \left( \frac{d \phi_4}{dt} - U_{4,5} \frac{d \phi_5}{dt} \right) - M_{c5} \tag{7}
\]

### III. CALCULATION RESULTS

The solution of the problem was made under the following values of the drive parameters, taking into account the motor power capacity, \( N \), voltage, \( U_v \), current frequency, \( f \), angular speed of the motor’s rotor, \( \omega_r \), rotational speed of the motor’s rotor, \( n_r \), the nominal torque moment, \( M_n \), the critical torque moment, \( M_c \), critical sliding, \( S_c \), and the moment of the rotor inertia – \( J_{rot} \).

The calculated moments of inertia for different mechanisms, in kgm\(^2\), are given below.

\[
J_{5pb} = 0.77 \cdot 10^{-3}; \quad J_4 = (J_{1cw} + J_{5pb}) = 1.0 \cdot 10^{-3}; \quad J_2 = 0.10 \cdot 10^{-3}; \quad J_3h = 0.41 \cdot 10^{-3}; \quad J_3s = 0.31 \cdot 10^{-3}; \quad J_{3n} = 0.67 \cdot 10^{-4};
\]

\[
J_{5sh} = 0.11 \cdot 10^{-3}; \quad J_{3bh} = 0.39 \cdot 10^{-4}; \quad J_4 = 0.29 \cdot 10^{-3}; \quad J_5h = 0.12 \cdot 10^{-3}; \quad J_{5sh} = 0.42 \cdot 10^{-4}; \quad J_{5sh} = 0.11 \cdot 10^{-3};
\]

\[
J_{5tb} = 0.15 \cdot 10^{-3}; \quad J_{5pb} = 0.62 \cdot 10^{-3}; \quad J_3 = (J_{5mh} + J_{5sh} + J_{3t} + J_{5sh}) = 0.29 \cdot 10^{-3};
\]

\[
J_5 = (J_{5sh} + J_{5pb} + J_{5h} + J_{5sh} + J_{5tm}) = 0.49 \cdot 10^{-3}.
\]

### IV. EXPERIMENT

In general, we were interested in research on changes in the spring (rubber) bushes rigidity of the driven pulleys in the sewing machine’s belt drives. The increased rigidities of the driven pulleys’ spring bushes allow some decreasing of the oscillation amplitude of angular speeds and loads on the driver shafts.

The rigidity factors, \( C_i \), were obtained with respect to the radius and elasticity of the segments in the mechanisms.

\[
C_1 = 600 \text{ Nm/rad}; \quad C_2 = 600 \text{ Nm/rad}; \quad C_3 = 600 \text{ Nm/rad}; \quad C_a = 260 \text{ Nm/rad};
\]

Dissipation of the belts, \( d_i \), was determined with respect to the cross sectional area of the belt and its length.

\[
d_1 = 5.4 \text{ Nm/s/rad}; \quad d_2 = 6.1 \text{ Nm/s/rad}; \quad d_3 = 5.2 \text{ Nm/s/rad}; \quad d_4 = 6.0 \text{ Nm/s/rad}.
\]

The ratio between the parts of the drive and the critical torque moment for the 3 and 5 components are given by the following values.

\[
U_{1,2} = 1.375; \quad U_{2,3} = 1.0; \quad U_{3,4} = 1.0; \quad U_{4,5} = 1.0.
\]

\[
M_{3c} = 0.175 \sin \omega 3t + (0.018...0.220) \text{Nm}; \quad M_{5c} = 0.14 \sin \omega 5t + (0.011...0.013) \text{Nm}.
\]

The solution was calculated taking into account the following initial conditions at \( t=0 \).

\[
\phi_1 = \phi_2 = \phi_3 = \phi_4 = \phi_5 = 0; \quad \dot{\phi}_1 = \dot{\phi}_2 = \dot{\phi}_3 = \dot{\phi}_4 = \dot{\phi}_5 = 0; \quad M_1 = 1.2 \text{Nm}; \quad M_{3c} = M_{5c} = 0.
\]

The considerable decrease of rigidity in the spring bushes’ pulleys results in an increase of the oscillation amplitude of angular speeds and loads on the shafts. The graphic regularities of the angular speed’s change and the fluctuations of acceleration and torque on the sewing machine’s bottom shaft are presented in Fig. 4.

![Fig. 4 a, b, c Regularities in the change of the angular speed, angular acceleration and torque under rigidity variation ratio for the split driven pulley’s spring](image-url)
bush of the toothed belt drive in the sewing machine

Fig. 5 a and b show the relationships between changes to the oscillation amplitude in speed, acceleration and torques for the bottom and main shafts under an increase of rigidity of the pulleys’ spring bushes.

Accordingly, the increase of rigidity of the driven pulley’s spring bush, which accelerates the belt drive, results in a decrease of the oscillation amplitude of speed, acceleration, and torque on the sewing machine’s main shaft as well.

The figures show that an increase of \( C_2 \) from 50 Nm/rad to 310 Nm/rad results in decreases of the angular speed’s amplitude of the bottom shaft from 84 s\(^{-1}\) to 25 s\(^{-1}\), the angular acceleration’s amplitude from 305 s\(^{-2}\) to 31 s\(^{-2}\), and the torque’s amplitude on the bottom shaft from 0.22 Nm to 0.045 Nm. It should be noted that amplitudes of \( \Delta \phi_3 \), \( \Delta \dot{\phi}_3 \) and \( \Delta M_3 \), as \( C_2 \) increases, has non-linear features too, but differs in the values, though the mean angular speed of both the bottom and main shaft is the same. Thus, with an increase of \( C_2 \) from 50 Nm/rad to 705 Nm/rad, the angular speed’s amplitude decreases from 115 s\(^{-1}\) to 28 s\(^{-1}\), the angular acceleration’s amplitude decreases from 310 s\(^{-2}\) to 34 s\(^{-2}\), and \( \Delta M \) decreases from 0.276 Nm to 0.051 Nm. This is explained by the fact that the main shaft makes complicated movements as well as, actually, all operating technological loads, including the resistances at motion and delivery of the bottom thread. For oscillation amplitudes less than \( \Delta \phi_3 = 60-70 \text{ s}^{-1} \) and \( \Delta \dot{\phi}_3 = 55-65 \text{ s}^{-1} \), the recommended values of the rigidities for the driven pulleys’ spring bushes of the sewing machine’s belt drives are \( C_2 = 195-215 \text{ Nm/rad} \) and \( C_3 = 190-205 \text{ Nm/rad} \). A further increase in the ratios of rigidity \( C_2 \) and \( C_3 \) results in an increase of the mean load values, which is not desirable.

At the same time, the mean value of \( M_5 \) increases from 0.07 – 0.08Nm, and \( M_3 \) decreases from 0.09 – 0.05Nm (see diagrams 1 and 2 on Fig. 5c).

The research, under the change of the belt drives’ parameters \((d_1, d_3, C_2, C_3)\), shows that with an increase of \( d_1 \) and \( d_3 \), the system transient time in the process increases considerably, and the load on the sewing machine’s drive shafts increases up to
10 - 15%. This is explained by the fact that the system operates (shafts’ rotation) under varying loads and oscillations in the angular speeds under which the absorption energy in the system’s spring elements is peculiar. Besides that, the significant increase of $C_1$ and $C_3$ is considered undesirable at the high rotational speeds of the main and bottom shafts of the sewing machine, as it can result in quick failure of the drive belts (enlarged belt deformation cycle). However, decreasing of the drive belts’ circular rigidities up to 230 - 240 Nm/rad results in an increase of the drives’ sliding along the drive pulleys. Therefore, the recommended parameter values of the sewing machine’s belt drives are

$$C_1 = 280 - 310 \text{Nm/rad}, \quad C_3 = 240 - 260 \text{Nm/rad}, \quad d_1 = 7.2 - 7.8 \text{Nms/rad}, \quad d_3 = 6.5 - 7.1 \text{Nms/rad}.$$  

V. CONCLUSION

The regularities in the rotation of the main and bottom shafts of the sewing machine were obtained during this research by the determination of the values for recommended parameters in the drive design. Moments of inertia for the different masses of the system have been defined with consideration of the factors of stiffness for the elastic belt tensioning roller and elastic (rubber) bush in the drive pulley for the belt transmission. Internal friction factors were taken into account at circular deformations of the hub in the drive pulley and the elastic belt transmission. The radiuses for the drive and the driven pulleys have been also observed in this study, as well as the shifting of the tensioning roller axis due to the deformation of the elastic bush. In addition, a new scheme with the split belt driven pulley and tensioning roller was developed to modernize the sewing machine’s drive.

The impact of the rigid-dissipative parameters in belt drives on the device’s motion mode was studied. The relationships in the change of the sewing machine’s main and bottom shafts’ load according to the change of the belt drives’ rigidity were also obtained. The recommended rigidity values of the parameters in the belt drives were established to be: $C_1 = (2.9 - 3.1) \cdot 10^2 \text{Nm/rad}, \quad C_3 = (2.4 - 2.6) \cdot 10^2 \text{Nm/rad}, \quad d_1 = (0.72 - 0.78) \cdot 10 \text{Nms/rad}, \quad d_3 = (0.65 - 7.1) \cdot 10 \text{Nms/rad}$.

REFERENCES